## Gravity - Connecting The Dots

Since the awakening of intellect, humans have been intrigued with connecting the dots of the unknown. Perhaps the most difficult puzzle: "What is gravity?" This paper is part of a series exploring this universal phenomenon and offers new insights into the mathematics involved along with a new analysis technique.

## History

Pythagoras of Samos (circa 571-495 BC) A Greek philosopher who observed characteristics of the right triangle that the square of the hypotenuse $\boldsymbol{c}$ is equal to the sum of the squares of the other two sides $\boldsymbol{a}, \boldsymbol{b}$. While it is argued that knowledge of this theorem predates him, he is credited with its first recorded proof.


This was an early demonstration that mathematics could be used explain the observed. More importantly was the concept of squares (exponential numbers).

Philolaus (circa 470-385 BC) A Greek Pythagorean and Pre Socratic philosopher argued that at the foundation of everything is the part played by the limiting and limitless, which combine together in a harmony. He is also credited with originating the theory "the earth is not the center of the universe".

Democritus (circa 460-370 BC) A Greek pre Socratic philosopher remembered most for his namesake for theories of government. His formulation of the atomic theory was an important breakthrough in theoretical thinking. His idea to divide things in half to a point where they cannot be cut any further gave us the word Atomos meaning "uncut-able".

Aristotle (circa 384-322 BC) A Greek philosopher and scientist who was concerned with things of the universe which he described as earth, water, air, fire and æther (precursors for solids, liquids, gas, plasma and the vacuum of space). He demonstrated an awareness of gravity with his idea that heavy objects fall faster than light ones in direct proportion to weight.

Archimedes of Syracuse (Circa 287-212 BC) An Ancient Greek mathematician, physicist, engineer, inventor, and astronomer. He is regarded as one of the leading scientists of classical antiquity. Archimedes anticipated modern calculus and analysis by applying concepts of infinitesimals and the method of exhaustion to derive and rigorously prove a range of geometrical theorems, including the area of a circle, the surface area and volume of a sphere, and the area under a parabola. His accomplishments marked the end of an early age of scientific discovery.

Dark ages (Circa 300 BC to 1480 AD) Little is known about any scientific breakthroughs during $1600+$ year period from Archimedes to Copernicus. Perhaps this was due to the fall of the Roman Empire or the domination of religious beliefs over scientific thinking during this time. What is interesting is Aristotle's mistaken ideas about gravity remained dogma for this entire period.

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Nicolas Copernicus (circa 1473-1543) A Polish renaissance mathematician and astronomer. His book De revolutionibus orbium coelestrium (On the Revolutions of Celestial Spheres) is considered a major event in the history of science, triggering a renaissance where Science re-emerged from the dark ages.

Even though scientific thinking was emerging, his ideas were in conflict with the teachings of the church. He formulated a model of the universe that placed the Sun rather than the earth as the center. Church censors expressed a desire to stamp out "Copernican doctrine".

Johannes Kepler (circa 1571-1630) A mathematician, astronomer, and astrologer. Gave us a first mathematical interpretation of the ideas postulated by Copernicus. He is best known for his laws of planetary motion set forth in his works Astronomia nova, Harmonices Mundi, and Epitome of Copernican Astronomy. These works became the foundations for later work by Sir Isaac Newton. Kepler offered a mathematical basis for connecting the dots.

Although he suggested "gravity" weakens as the inverse of the distance he must have had some insight into the nature of gravity as being related to parts of a mass rather than the whole. Kepler's laws improved the model of Copernicus. If the eccentricities of the planetary orbits are taken as zero, then Kepler basically agrees with Copernicus:

1. The orbit of every planet is an ellipse with the Sun at one of the two foci.
2. A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

He spent some time working out possible mathematical solutions to this problem including the Kepler Conjecture where he filled space with spheres, perhaps as a way to calculate the attraction between each.

Galileo Galilei (Circa 1564-1642) An Italian astronomer, physicist, engineer, philosopher, and mathematician who played a major role in the scientific revolution during the Renaissance.

His experiment dropping balls of the same size but different masses from the Leaning Tower of Pisa. This demonstrated their time of descent was independent of their mass, contrary to what Aristotle had taught. Galileo proposed that a falling body would fall with a uniform acceleration, as long as the resistance of the medium through which it was falling remained negligible. The effects of mass and inertia exactly balance and so the heavier mass reaches the ground at the same time.

It should be noted that even 100 years after Copernicus, Galileo was tried by the inquisition and found "vehemently suspect of heresy", forced to recant, and spent the rest of his life under house arrest. His ideas about gravity were developed using a "flat earth" model.

Isaac Newton (Circa 1643-1727) An English physicist and mathematician. His book Philosophiæ Naturalis Principia Mathematica ("Mathematical Principles of Natural Philosophy") published in 1687, laid the foundations for classical mechanics.

His laws were the beginning of understanding of inertia, acceleration and gravity. He shares credit with Gottfried Leibniz for the development of calculus.

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## Newton's laws:

First - When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force.

Second - When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body. The vector sum of the external forces F on an object is equal to the mass m of that object multiplied by the acceleration vector of an object. This is shown in his equation:

## $\mathrm{F}=\mathrm{ma}$

Third - All objects attract each other with a force of gravitational attraction. This force of gravitational attraction is directly dependent upon the masses of both objects and inversely proportional to the square of the distance that separates them. Newton formalized this in his equation:

$$
F=G(m 1 \mathrm{~m} 2) / D^{2}
$$

The inverse square law states a specified physical quantity or intensity is inversely proportional to the square of the distance from the source of that physical quantity.

Newton's law of universal gravitation follows an inverse-square law, as do the effects of electric, magnetic, light, sound, and radiation phenomena.

Carl Friedrich Gauss (Circa 1777-1885) A German mathematician contributed to many fields including algebra, differential geometry and geophysics.

He is remembered for his law for flux: The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

He extended it with a similar law for gravity: The gravitational flux through any closed surface is proportional to the enclosed mass.

Although his law for gravity is equivalent to Newton's, in many situations Gauss's law offers a simpler solution.

While both Newton and Gauss used shortcuts to combine all the mass bounded in a sphere as the basis for calculation for "total gravity", Gauss recognized the gravitational field $\mathbf{g}$ (also called gravitational acceleration) is a field with tensors to each point of space.

He concept was "the gravitational force experienced by a particle is equal to the mass of the particle multiplied by the gravitational field at that point". He then went on to apply his "hollow sphere" theorem which did not accomplish that.

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## New Observations

By the end of the 19th century, it was known that Mercury's orbit showed slight perturbations that could not be accounted for entirely under Newton's theory, but all searches for another perturbing body (such as a planet orbiting the Sun even closer than Mercury) had been fruitless.

Later, Newton's third law does not explain discrepancies observed by NASA that various spacecraft have experienced greater acceleration than expected during gravity assist maneuvers.

Perhaps if Newton had increased accuracy in measurement of distances planets traveled or time he would have corrected his laws to explain these phenomena. Further, our understanding of the underlying causes of gravity may have been clearer.

This points out the need to have the tools to measure what we observe before conjecturing on what it is that we are trying to explain.

For a fresh perspective, harken back to the wisdom of Democritus where things (mass) could be divided until they could be divided no further - these smallest of matter (atoms) still had mass. Couple this with Aristotle who thought that objects had attraction to the earth. Add Kepler's conjectures about orbiting objects and followed by Galileo who showed that all objects fall at the same rate regardless of their mass.

These observations show that taking a single object and splitting it into two halves and dropping them side by side they would fall at the same speed as each other. Further, if they were screwed together there won't be any force on the screws (side by side). However, if the two pieces were dropped in tandem, the forward one has more attraction pulling the rear one. This would put tension on the screws.

The effects of this are shown on earth as tidal forces or different weights measured at different elevations or more dramatically by the breakup of the Comet Shoemaker-Levy 9 that hit Jupiter in July 1994.


Notice the larger objects near the center. This is a result of the breakup of the original sphere. This may offer clues on a scalar mathematical representation of gravitational forces.

These insights lead to a possible better understanding of Newton's third law (as refined by Gauss).

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## New Math

One school of thought is that the issue of Mercury's orbit was resolved in 1915 by Albert Einstein's theory of general relativity.

A simpler explanation which possibly explains the Mercury orbit discrepancy (plus all outer planets) and NASA's slingshot anomalies is found in a review of the math of Newton and Gauss. They both saw the problem as attraction between the "masses comprising an object".

In an effort to simplify the problem, the mathematical solutions they offered were reduced to "summing the masses in each body before calculating the relative attraction between them" Versus "calculating the attraction of individual masses between each body before summing them." Restating Newton's third law to reflect this nuance:

The force of gravitational attraction is directly dependent upon the product of the sum of the attraction of individual points of mass contained in two objects and inversely proportional to the square of the distance between them.

This has interesting consequences.
First, the center of gravity for calculating each body's attraction in orbital relationships changes because of the differences in forces due to the inverse square making closer mass points having a higher attraction than distant mass points, thus centers of gravitational attraction are closer than the physical centers.

Second, the diameter of each body comes into play. The forces are not a single vector but become tensors from all individual masses of one body to all individual masses of the other body(s).


As can be seen the math is quite complex. Each mass particle has multiple attractions to other mass particles in each body

While it may be possible to reduce the forces between each body to a scalar value relating to distance between the centers with calculus, attempts of simplification by Newton and Gauss have fallen short. I propose a solution using Finite element analysis (FEA) as the basis for approximation. The number of matrix elements can be expanded as accuracy dictates and computing capability allows.

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## Mathematical example

The following shows static attraction using Newton's formulas applied to Bodies as single masses:

| Parameters |  |  |  |
| :--- | :--- | :--- | ---: |
| Diameter | Body A | A | 1,000 |
|  | Body B | B | 1,000 |
| Mass | Body A | $\mathbf{M}_{\mathrm{a}}$ | 2 |
|  | Body B | $\mathbf{M}_{\mathbf{b}}$ | 2 |
| Distance A B Cen |  | $\mathbf{D}$ | 10,000 |
| Attraction | $\mathbf{G}$ | 1 |  |

Figure 1 - Single mass at centers


| Seg | Cen | D | $\mathrm{F}=\mathrm{G}\left(\mathbf{M}_{\mathrm{a}} \mathbf{M}_{\mathrm{b}}\right) / \mathrm{D}^{2}$ | Cen |
| :---: | :---: | :---: | ---: | :---: |
| $\mathbf{1}$ | $\mathrm{Ma}_{1}$ | $10,000.0000$ | 0.0000000400000000 | $\mathbf{M b}_{\mathbf{1}}$ |
| Inverse square sum of forces $>$ |  | $\mathbf{0 . 0 0 0 0 0 0 0 4 0 0 0 0 0 0 0 0}$ |  |  |

The following show static attraction using revised formulas applied to Bodies as multiple mass points:
Parmaeters

| Mass | Body A | $\mathbf{M}_{\mathbf{a}}$ |
| :--- | :--- | :--- |
| Body B | $\mathbf{M}_{\mathrm{b}}$ | 1 |
|  |  | 1 |

Figure 2 - Split mass at centers


| Seg | Cen | D | $\mathrm{F}=\mathrm{G}\left(\mathrm{M}_{\mathrm{a}} \mathrm{M}_{\mathrm{b}}\right) / \mathrm{D}^{2}$ | Cen |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Ma ${ }_{1}$ | 10,000.0000 | 0.0000000100000000 | $\mathrm{Mb}_{1}$ |
| 2 | $\mathrm{Ma}_{2}$ | 10,000.0000 | 0.0000000100000000 | $\mathrm{Mb}_{2}$ |
| 3 | $\mathrm{Ma}_{2}$ | 10,000.0000 | 0.0000000100000000 | $\mathrm{Mb}_{1}$ |
| 4 | Ma ${ }_{1}$ | 10,000.0000 | 0.0000000100000000 | $\mathrm{Mb}_{2}$ |
| Inverse square sum of forces |  |  | $\mathbf{0 . 0 0 0 0 0 0 0 4 0 0 0 0 0 0 0 0 ~}$ |  |

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Figure 3 - Move split masses to quarters


| Seg | Cen | $\mathbf{D}$ | $\mathbf{F}=\mathbf{G}\left(\mathbf{M}_{\mathbf{a}} \mathbf{M}_{\mathbf{b}}\right) / \mathbf{D}^{\mathbf{2}}$ | $\mathbf{C e n}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{M a}_{1}$ | 10000.0000 | 0.0000000100000000 | $\mathbf{M b}_{\mathbf{1}}$ |
| $\mathbf{2}$ | $\mathbf{M a}_{\mathbf{1}}$ | 10049.8756 | 0.0000000099009901 | $\mathbf{M b}_{\mathbf{2}}$ |
| $\mathbf{3}$ | $\mathbf{M a}_{\mathbf{2}}$ | 10049.8756 | 0.0000000099009901 | $\mathbf{M b}_{\mathbf{1}}$ |
| $\mathbf{4}$ | $\mathbf{M a}_{\mathbf{2}}$ | 10000.0000 | 0.0000000100000000 | $\mathbf{M b}_{\mathbf{2}}$ |
| Inverse square sum of forces $\boldsymbol{>}$ |  |  | $\mathbf{0 . 0 0 0 0 0 0 0 3 9 8 0 1 9 8 0 2}$ |  |

Figure 4 - Move masses to aphelion, perihelion


| Seg | Cen |  | D | $\mathrm{F}=\mathrm{G}\left(\mathrm{M}_{\mathrm{a}} \mathrm{M}_{\mathrm{b}}\right) / \mathrm{D}^{2}$ | Cen |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Ma ${ }_{1}$ |  | 10,000.0000 | 0.0000000100000000 | $\mathrm{Mb}_{1}$ |  |
| 2 |  | Ma ${ }_{2}$ | 10,000.0000 | 0.0000000100000000 |  | $\mathbf{M b}$ |
| 3 |  | Ma | 9,000.0000 | 0.0000000123456790 | $\mathrm{Mb}_{1}$ |  |
| 4 | Ma ${ }_{1}$ |  | 11,000.0000 | 0.0000000082644628 |  | $\mathrm{Mb}_{2}$ |
| Inverse square sum of forces > |  |  |  | $\mathbf{0 . 0 0 0 0 0 0 0 4 0 6 1 0 1 4 1 8}$ |  |  |

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Figure 5 -Mass at Quadrants



Hoc est non in finem. Optimum est tamen ad venit.

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